1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 4, Module 5, Problem based on Work-Energy Theorem Chapter 6, Work, Energy and Power
Module Id	Keph_10605_eContent
Pre-requisites	Kinematics, laws of motion, basic vector algebra ,work energy theorem conservative and non conservative forces
Objectives	After going through this module, the learners will be able to: • Use work energy theorem for simple problem solving • Analyze Motion in a vertical circle
Keywords	Work energy theorem, potential energy, kinetic energy, motion in a vertical circle

2. Development Team

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1. UNIT SYLLABUS

UNIT IV: Chapter 6: WORK ENERGY AND POWER

Work done by a constant force and a variable force; Kinetic energy; Work energy theorem; power.

Notion of potential energy; potential energy of a spring conservative and non conservative forces; conservation of mechanical energy (kinetic and potential energies) non-conservative forces; motion in a vertical circle; Elastic and inelastic collisions in one and two dimensions.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS — 7 Modules

This unit is divided into 7 modules for better understanding.

Module 1	• Meaning of work in the physical sense • Constant force over variable displacement • variable force for constant displacement • Calculating work • Unit of work • Dot product • Numerical
Module 2	• Different forms of energy • Kinetic energy • Work energy theorem • Power
Module 3	• Potential energy • Potential energy due to position • Conservative and non conservative forces • Calculation of potential energy
Module 4	• Potential energy • Elastic Potential energy • Springs • Spring constant • problems
Module 5	• Motion in a vertical circle • Applications of work energy theorem • Solving problems using work power energy
Module 6	• Collisions • Idealism in Collision in one dimension • Elastic and inelastic collision • Derivation
Module 7	• Collision in two dimension • Problems

Module 5

3. WORDS YOU MUST KNOW

Let us keep the following concepts in mind

- **Rigid body:** An object for which individual particles continue to be at the same separation over a period of time.
- **Point object**: **Point object** is an expression used in kinematics: it is an **object** whose dimensions are ignored or neglected while considering its motion.
- **Distance travelled**: change in position of an object is measured as the distance the object moves from its starting position to its final position. Its SI unit is m and it can be zero or positive.
- **Displacement**: a **displacement** is a vector whose length is the shortest distance from the initial to the final position of an object undergoing motion. . Its SI unit is m and it can be zero, positive or negative.
- **Speed**: Rate of change of position .Its SI unit is ms^{-1}.
- **Average speed=:** $\frac{\text{total path length travelled by the object}}{\text{total time interval for the motion}}$

  Its SI unit is ms^{-1}.
- **Velocity (v)**: Rate of change of position in a particular direction.
  Its SI unit is ms^{-1}.
- **Instantaneous velocity**: velocity at any instant of time.

$$v_{instaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

  Instantaneous velocity is the **velocity** of an object in motion at a specific time. This is determined by considering the time interval for displacement as small as possible .the instantaneous velocity itself may be any value .If an object has a constant **velocity** over a period of time, its average and **instantaneous velocities** will be the same.
- **Uniform motion**: a body is said to be in uniform motion if it covers equal distance in equal intervals of time

- **Non uniform motion**: a body is said to be in non- uniform motion if it covers unequal distance in equal intervals of time or if it covers equal distances in unequal intervals of time
- **Acceleration (a)**: time rate of change of velocity and its SI unit is ms^{-2}. Velocity may change due to change in its magnitude or change in its direction or change in both magnitude and direction.
- **Constant acceleration**: Acceleration which remains constant throughout a considered motion of an object
- **Momentum (p):** The impact capacity of a moving body. It depends on both mass of the body and its velocity. Given as $p = mv$ and its unit is kg ms^{-1}.
- **Force** (F): Something that changes the state of rest or uniform motion of a body. SI Unit of force is Newton (N). It is a vector, because it has both magnitude, which tells us the strength or magnitude of the force and direction. Force can change the shape of the body.
- **Constant force**: A force for which both magnitude and direction remain the same with passage of time
- **Variable force**: A force for which either magnitude or direction or both change with passage of time
- **External unbalanced force**: A single force or a resultant of many forces that act externally on an object.
- **Dimensional formula**: An expression which shows how and in which way the fundamental quantities like, mass (M), length (L) and time (T) are connected
- **Kinematics:** Study of motion of objects without involving the cause of motion.
- **Dynamics**: Study of motion of objects along with the cause of motion.
- **Vector:** A physical quantity that has both magnitude and direction .displacement, force, acceleration are examples of vectors.
- Vector algebra: Mathematical rules of adding, subtracting and multiplying vectors.
- Resolution of vectors: The process of splitting a vector into various parts or components. These parts of a vector may act in different directions. A vector can be resolved in three mutually perpendicular directions. Together they produce the same effect as the original vector.

- Dot product: If the product of two vectors (A and B) is a scalar quantity. Dot product of vector A and B: A.B= $|A||B|cos\theta$ where θ is the angle between the two vectors

  Since Dot product is a scalar quantity it has no direction. It can also be taken as the product of magnitude of A and the component of B along A or product of B and component of A along B.

- Work: Work is said to be done by an external force acting on a body if it produces displacement W= F.S cosθ, where work is the dot product of vector F(force) and Vector S (displacement) and θ is the angle between them . Its unit is joule and dimensional formula is ML^2T^{-2}. It can also be stated as the product of component of the force in the direction of displacement and the magnitude of displacement. Work can be done by constant or variable force and work can be zero, positive or negative.

- Energy: The ability of a body to do work

- Kinetic Energy: The energy possessed by a body due to its motion = ½ mv^2, where 'm' is the mass of the body and 'v' is the velocity of the body at the instant its kinetic energy is being calculated.

- Work Energy theorem: Relates work done on a body to the change in mechanical energy of a body i.e.,

  $W = F.S = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_i^2$

- Conservative force: A force is said to be conservative if the work done by the force in displacing a body from one point to another is independent of the path followed by the particle and depends on the end points. Example: gravitational force.
- Non- conservative forces: If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. Example: friction.

- Conservation of mechanical energy: Mechanical energy is conserved if work done is by conservative forces.
- Potential energy due to position: Work done in raising the object of mass m to a particular height (distance less than radius of the earth) = m g h.

4. INTRODUCTION

We have being considering work done by a force in physics. This we have seen is measurable and we give it as a scalar product of vector F and vector S.

We have learnt, that the force being constant must be causing a change in velocity of a body. As we had considered in previous modules, this would result in change in kinetic energy.
Work energy theorem relates the amount of work done by a force causing displacement of a body to the change in the kinetic energy of the body.
We can extend this idea to change in potential energy as in the case of objects changing altitude above a reference level or change in configuration as in the case of springs .
We, however, maintained that for the relation to be simple the work must be done only by conservative forces such as gravitational force. In such cases: total mechanical energy of a system is conserved.

We have checked the above statement mathematically in module 3 and in module 4.
In the example of an object falling under gravity where we could calculate the potential and kinetic energies at different heights above a chosen reference horizontal level.
We also considered the changes in kinetic and potential energies in a simple pendulum in motion.

http://www.physicsclassroom.com/mmedia/energy/pe.cfm

Both cases illustrated conservation of mechanical energy so long as the forces acting on the system are conservative.
If some of the forces involved are non conservative, part of the mechanical energy may get transformed into other forms such as heat, light and sound .However, the total energy of an isolated system does not change as long as we account for all forms of energy.

Energy may be transformed from one form to another but the total energy of an isolated system remains constant Energy can neither be created, nor destroyed.

WHAT IS AN ISOLATED SYSTEM?

We define a system as a collection of two or more objects.

An isolated system, is a system in which the only forces that contribute to the momentum change of an individual object are the forces acting between the objects themselves, and no external unbalanced force is responsible for it.

Since the universe, as a whole may be viewed as an isolated system, the total energy of the universe is constant. If one part of the universe loses energy, another part must gain an equal amount of energy.

The principle of conservation of energy cannot be proved. However, no violation of this principle has been observed.

The concept of conservation and transformation of energy into various forms links together various branches of science physics, chemistry and life sciences. It provides a unifying, enduring element in our scientific pursuits. From engineering point of view, all electronic communication and mechanical devices rely on some forms of energy transformation.

In this module we will take up some interesting examples and use work energy theorem to find solutions.

5. APPLICATIONS OF WORK-ENERGY THEOREM AND CONSERVATION OF ENERGY

We can make use of the above principle to easily solve problems that look complex Here are some examples:

EXAMPLE:

To calculate minimum stopping distance for a vehicle:

When we **apply brakes in a car, moving at a particular speed, say *v*, in order to stop it.**

- **How does it stop?**

- **Why does it stop?**
- **What happens to its kinetic energy?**
- **Why does it stop after moving a certain distance?**

Negative work done, by the force of friction, acting between the tyres of the car and the road, reduces its kinetic energy to zero.

Let μ be the coefficient of friction and *v* be the velocity of the car just before the brakes were applied.

If the car stops after covering a distance *d* - called stopping distance

Let us use work-energy theorem

Friction force = $\mu m g$

Work done by friction force in stopping the car after distance (*displacement* = *distance*) d

= **- F. d**

= $-\mu mg.d$

This is equal to the total kinetic energy if we ignore heat, sound energies

$-\mu mg.d = \frac{1}{2}mv^2$

So stopping distance **d** = $\frac{\frac{1}{2}v^2}{\mu g}$

This gives *d*, the minimum stopping distance for the car

From the above result we have points to ponder:

Carefully read and think about these

1. **Stopping distance for a vehicle** = $\frac{\text{kinetic energy}}{\text{stopping force}}$

2. Stopping distance for a vehicle is determined by the effective coefficient of friction between the tyres and the road.

3. Stopping distance does not depend on the mass of the vehicle to be stopped. It means that a truck and a car will stop over the same distance.

4 Stopping distance (d) is proportional to the square of the speed (v^2).

5. If two vehicles, of masses m_1 and m_2, moving at velocities v_1 and v_2, are stopped by applying the same braking force, their stopping distances will be directly proportional to their respective kinetic energies.

6. If two vehicles, moving with the same kinetic energy, are stopped by applying the same braking force, they will stop over the same distance irrespective of their masses or velocities.

7. If two vehicles are stopped by applying the same braking force, their stopping times will be in the ratio of their momenta.

ANSWER THE FOLLOWING:

1. **A bus and a car, moving with the same kinetic energy, are brought to rest by applying brakes which provide the same stopping force. Which one of them will come to a stopover a shorter distance?**
2. **When the brakes of a car, going at 50 km/h are applied to stop the car after a distance of d m, how much farther will it move, if the brakes are applied to the car moving at 80 km/h?**
3. **Two persons, of equal weight, are running at 4 m/s and 5 m/s respectively. Both increase their speed by 1 m/s in a time span of 10 s. Who does more work? Who develops more power?**

6. MOTION IN A VERTICAL CIRCLE

At a Mela or at a fair we see giant wheels.

https://www.youtube.com/watch?v=IpCJnoiJPFI

Giant wheel in Gurgaon

https://www.google.co.in/search?site=imghp&tbm=isch&q=giant+wheel&tbs=sur:fmc&gws_rd=cr&ei

In amusement parks, and bikers or cyclists looping the 'death well' in a circus, are some of the better known examples of motion in a vertical circle.

http://www.physicsclassroom.com/mmedia/energy/ce.cfm

See the video of a bucket of water being swirled in a vertical circle

http://www.youtube.com/watch?v=baQrVTTm-hU

You might have seen or heard about a pilot doing acrobatics at an air show , taking the plane in a vertical loop

https://www.youtube.com/watch?v=H4MT5_Gt6TU

Aircraft acrobatics

Or **simply a stone tied to a string and swirled in a vertical circle**

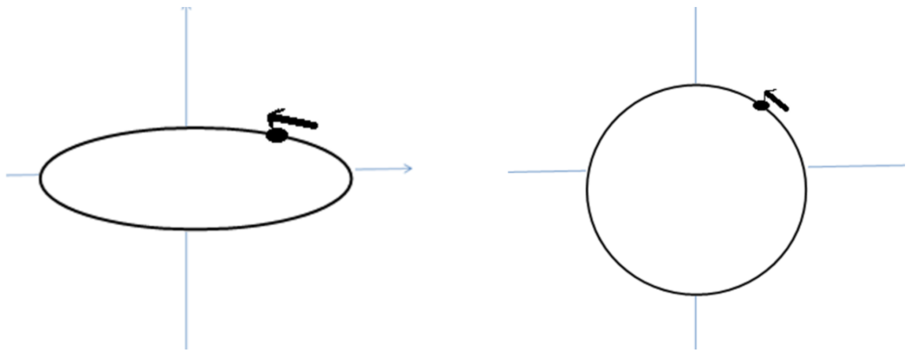
Motion in a vertical circle is different from that in a horizontal circle.

Let us try to understand the physics behind the motion of an object in a vertical circle.

Motion in a vertical circle is different from that in a horizontal circle, because not only do the speed and the direction of motion change continuously, but the force of gravity also plays an important role in this kind of a circular motion.

The fig. shows a body moving in a circle – horizontal and vertical

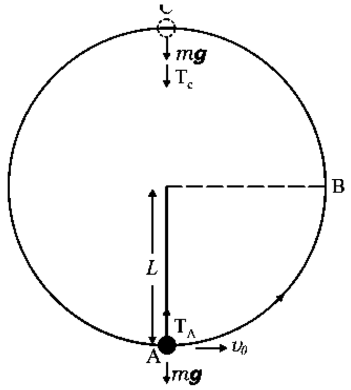
We are restricting our choice of plane of motion to only horizontal and vertical



Notice:

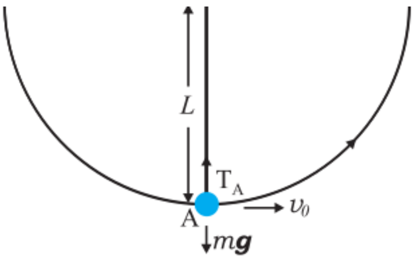
One in vertical circle and the second in a horizontal circle by this we mean the circular track is in a vertical plane or it is in a horizontal plane.

Draw a circle on a paper place it horizontally on a flat table, next hold it up vertically this is the picture you must imagine.



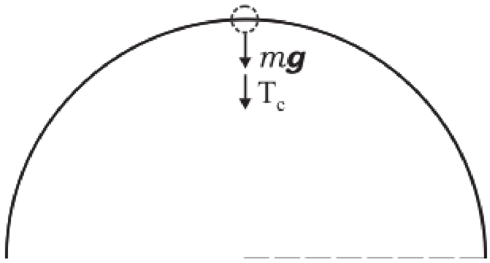
The diagrams show the motion in a vertical plane, say it is a bob tied to a string.

At the lowest point of the vertical circl



$$F_{net} = mg - T_A$$

At the highest point of the vertical circle



$$F_{net} = mg + T_C$$

There are two external forces on the bob

i) Gravity and

ii) The tension in the string.

The later does no work since the displacement of the bob is always normal to the string.

The potential energy of the bob is thus associated with the gravitational force only.

The total mechanical energy E of the system is conserved.

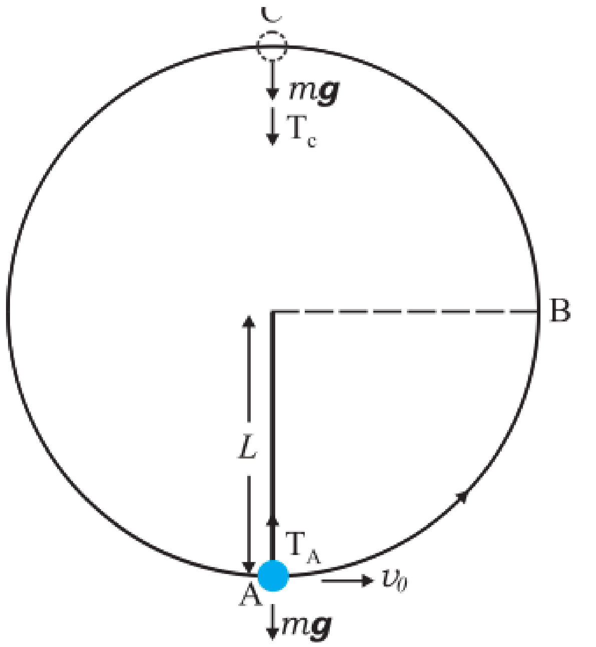
POINTS TO REMEMBER:

- **The value of radial acceleration changes.**
- **A object such as a stone / bucket of water whirled in a vertical circle has tension in the string or hand pushing it towards the centre tending to make the object fall at the highest point.**
- **It is different from motion in a horizontal plane.**
- **It is a case of non - uniform motion.**
- **The speed of the object is not the same .the speed decreases on the way up say from point A to C and increases on its way down from C to A.**
- **The object has a radial acceleration towards the centre as long as it moves in a circle.**

We can use mathematics if we assign values to a situation and use equations to solve our problem.

EXAMPLE:

A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A, such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C.



Obtain an expression for:

(i) v_0

(ii) The speeds at points B and C.

(iii) The ratio of the kinetic energies (K_B / K_C) at B and C.

Comment on the nature of the trajectory of the bob after it reaches the point C.

SOLUTION:

Remember

There are two external forces on the bob: gravity and the tension (T) in the string. The later does no work since the displacement of the bob is always normal to the string. The potential energy of the bob is thus associated with the gravitational force only. The total mechanical energy E of the system is conserved.

We take the potential energy of the system to be zero at the lowest point A and that it has only kinetic energy due to its speed v_0.

Thus,

$E_A = \frac{1}{2} m v_0^2$ ------------------- (1)

And to keep it moving in a circle

$T_A = mg = \frac{m v_0^2}{L}$ ------- (2)

Where, T_A **is the tension in the string at A.**

At the highest point C, the string slackens, as the tension in the string (T_C) becomes zero. Also energy is not only kinetic energy but also potential energy.

Work done in raising the bob to a height of 2L = mg 2L = 2m g L

E_C = **KE + PE** = $\frac{1}{2}mv_C^2 + 2mgL$ --------- (3)

For the tension in the string at the highest point C (T_C) becomes zero or it just slackens which means the bob will no longer move in a circle and fall.

$mg = \frac{mv_C^2}{L}$ ----------------- (4)

$E_C = \frac{5}{2}mgL$ - ------------------ (5)

$E_C = E_A$

$$\frac{1}{2}mv_0^2 = \frac{5}{2}mgL$$

$$v_0 = \sqrt{5gL}$$

This is the minimum speed required for the bob tied to a string of length L to complete the vertical circle.

If the speed is less than $v_0 = \sqrt{5gL}$, the bob will fall on approaching the highest point.

We can use this general relation to find the tension in the string for different positions of the bob in its (vertical) circular path.

To consolidate the above steps for derivation:

1. **Consider centripetal force at the location**

2. **At highest point tension should be positive**
3. **Use conservation of energy for velocity at any location**

ii) Speed at C

$V_c = \sqrt{gL}$

To find speed at B we use work-energy theorem

The energy at B

$E_B = \frac{1}{2}mv_B^2 + mgL$

This is equal to the total energy at A

$\frac{1}{2}m{v_0}^2 = \frac{5}{2}mgL$

Or $\frac{1}{2}mv_B^2 + mgL = \frac{5}{2}mgL$

$V_B = \sqrt{3gL}$

iii) The ratio of the kinetic energies (K_B / K_C) at B and C.

$$\frac{\mathbf{K_B}}{\mathbf{K_C}} = \frac{\frac{1}{2}\mathbf{m_B}^2}{\frac{1}{2}\mathbf{m_C}^2} = \frac{\mathbf{3}}{\mathbf{1}}$$

At point C, the string becomes slack and the velocity of the bob is horizontal and to the left.

If the connecting string is cut at this instant, the bob will execute a projectile motion with horizontal projection akin to a rock kicked horizontally from the edge of a cliff. Otherwise the bob will continue on its circular path and complete the revolution.

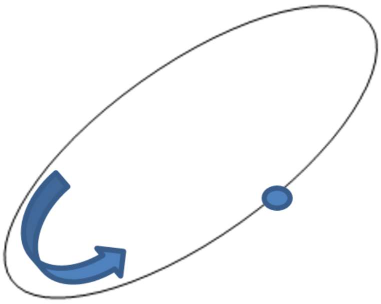
Think about this:

Consider the circle that you had drawn on a paper and had made it lie in a:

i) Horizontal

ii) Vertical plane

If the circular path is in a plane inclined to the horizontal.



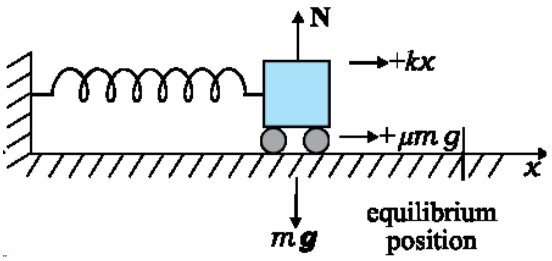
- **Is the motion uniform or non-uniform?**
- **How would you determine the speed, kinetic energy of a bob at any point in the track?**

EXAMPLE:

To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant $6.25 \times 10^3 Nm^{-1}$. What is the maximum compression of the spring?
Suppose the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring

SOLUTION

In presence of friction, both the spring force and the frictional force act so as to oppose the compression of the spring as shown in the figure.



We use the work-energy theorem, rather than the conservation of mechanical energy.

The change in kinetic energy is

$\Delta \mathbf{K} = \mathbf{K_f} - \mathbf{K_i}$

$= 0 - \frac{1}{2}mv^2$

The work done by the net force is

$\mathbf{W = -\frac{1}{2}kX^2{}_m - \mu mgX_m}$

On equating the above two equations

We have

$$\mathbf{-\frac{1}{2}mv^2 = -\frac{1}{2}kX^2{}_m - \mu mgX_m}$$

Calculating after putting values we have:

X_m = 1.35 m, which is less than our earlier calculated value of 2.0m in module 4.

We consider a situation in which the two forces on the body consist of a conservative force F_c and a non-conservative force F_{nc}, the conservation of mechanical energy formula will have to be modified in this situation.

$\mathbf{(F_c + F_{nc})\,\Delta x = \Delta K}$

Solving this problem would have been lot more complicated had we not just used work energy theorem

$\mathbf{E_f - E_i = W_{nc}}$

where **W_{nc} is the total work done by the non-conservative forces over the path**.

Note that unlike the conservative force, W_{nc} depends on the particular path from initial position to final position.

7. ENERGY REQUIRED TO FORM A CLOUD

Have you ever wondered how much Energy would be needed to lift water to make a cloud?

https://www.goodfreephotos.com/cache/portugal/other-portugal/mountains-sky-road-and-clouds-in-poland_800.jpg?cached=1533218238

Suppose -A group of clouds at a height of 500 m above the ground bursts and rainfall covers an area of 10^6 m^2 with a depth of 2 cm.

How much work would be done in raising water to a height to make a cloud?

SOLUTION

volume of water raised $= 10^6 \times 2 \times 10^{-2} = 2 \times 10^4 \mathrm{m}^3$

Mass of water = volume $\times$ density of water

$$= 2 \times 10^4 \times 10^3 kg$$

Work done in raising this mass to a height of 500m is:

$mgh = 2 \times 10^4 \times 10^3 kg \times 10 ms^{-2} \times 500m$

$= 10^{11}$J

Now that is a lot of energy.

Ever wondered where this energy comes from? Water needs energy from the atmosphere to evaporate!

8. SOLVED EXAMPLES

EXAMPLE:

Two identical balls of mass m are thrown from a window of a multistory building, at a height h above the ground Initial speed of each ball is the same, but are thrown in different directions and at different angles. What is the speed of each ball as it strikes the ground?

SOLUTION:

Each ball is subjected to gravitational force mg after it is thrown

For each ball the initial potential energy with respect to the ground is m g h

Initial KE of both balls is the same

Balls will land with the same speed

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgh$$

Of course we have ignored any air resistance.

Also, the balls may not strike the ground at the same time

EXAMPLE:

In a pole vault, an athlete uses a pole to convert the kinetic energy of running into potential energy when the pole is vertical. A good sprinter runs at a speed of 10 m/s. How high can the athlete raise his center of gravity?

SOLUTION

https://www.goodfreephotos.com/vector-images/pole-vaulter-vector-clipart.png.php

Initially say the centre of gravity of the athlete is 1 m above the ground

Initial PE = 0

Initial Kinetic energy = ½ mv^2

At the top instantaneous final velocity = 0 final P.E. = m g H, because only force of gravity acts on the airborne sprinter.

$$\mathbf{H = \frac{v^2}{2g} = \frac{(10ms^{-2})^2}{2 \times 10m} = 5m}$$

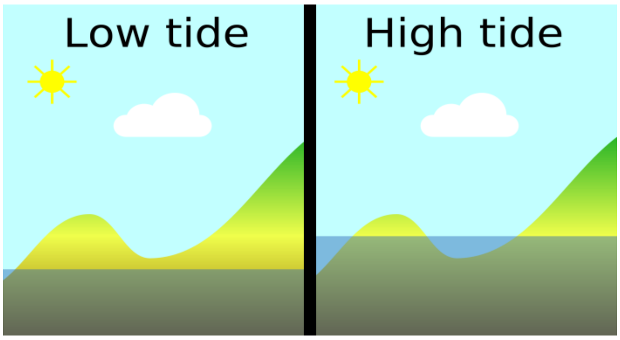
Adding the value of initial point of lowest reference potential energy at 1m

So corrected H will be: 1m + 5 m = 6 m from the ground

EXAMPLE:

Tides are used to generate electrical energy at a dam across the mouth of Rance River in France. At a location the tides rise to 8.5 m above low tide. The river basin is closed off after it fills at high tide and at low tide the water is allowed to fall on turbines to generate electricity.

SOLUTION:



The area of the basin is 23 km square. How much work does the falling water do? Neglect initial and final kinetic energy assuming them to be the same.

Mass of water = density × volume

Average drop in level may be taken as ½ H because once the water starts to flow out of the basin.

Work done by applied force = P.E. final – P.E. initial = - $\frac{mg\,H}{2}$

Work is negative because water does work on the turbine.

Some energy is Lost -as sound energy.

This facility can be installed in India in Gujarat but the cost or making the reservoir is very high

9. SUMMARY

So in this module you have learnt:

- Application of work- energy theorem and conservation of mechanical energy to solve simple problems.
- You analyzed motion in a vertical circle because it is different from motion in a horizontal circle.
- You used work- energy theorem to find out the amount of energy needed to form clouds, the height an athlete would rise for pole-vault, why a run up is necessary and how do tides give energy for electricity generation